# Lecture 11: The sticky-price monetary model cont'd.

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- Note typo in lecture 1: Equation 39 should read

$$F'(K_2) = r$$

# Solving systems of two differential equations: general case

- Simple homogenous linear system

$$\begin{aligned} \dot{x_1} &= a_{11}x_1 + a_{12}x_2 \\ \dot{x_2} &= a_{21}x_1 + a_{22}x_2 \end{aligned}$$

- Stationary equilibrium: $(x_1 = 0, x_2 = 0) \rightarrow \dot{x_1} = \dot{x_2} = 0$

- Define

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

- Recall that

$$\begin{aligned} tr(A) &= a_{11} + a_{22} \\ |A| &= a_{11}a_{22} - a_{12}a_{21} \end{aligned}$$

- Definition of stability:

  – The system is stable if for arbitrary initial values $x_1$ and $x_2$ tend to the stationary equilibrium as $t \longrightarrow \infty$

  – The system exhibits *saddle path stability* if there is a unique convergent path to the steady state. That is; if the initial value of one of the variables is given, there is a unique initial value of the other variable consistent with the system converging to the stationary equilibrium as $t \longrightarrow \infty$

- Mathematical conditions for stability

  – Iff $tr(A) < 0$ and $|A| > 0$ the system is stable

  – If $|A| < 0$ the system exhibits *saddle path stability*

- Consider now the non-linear system

$$
\begin{aligned}
\dot{y_1} &= \phi_1(y_1, y_2) \\
\dot{y_2} &= \phi_2(y_1, y_2)
\end{aligned}
$$

- The *Jacobian* of the system is defined by

$$
A = \begin{bmatrix} \frac{\partial \phi_1}{\partial y_1} & \frac{\partial \phi_1}{\partial y_2} \\ \frac{\partial \phi_2}{\partial y_1} & \frac{\partial \phi_2}{\partial y_2} \end{bmatrix}
$$

- Conditions for (local) stability and saddle-point stability are the same as above except that we **replace the coefficient matrix with the Jacobian evaluated at the stationary state** $(\overline{y}_1, \overline{y}_2)$ **defined by** $\phi_1(\overline{y}_1, \overline{y}_2) = \phi_2(\overline{y}_1, \overline{y}_2) = 0$

# Solving the Dornbusch model

- Model

$$Y = C(Y) + X\left(\frac{SP^*}{P}, Y, Y^*\right) \tag{1}$$

$$\frac{M}{P} = m(i, Y) \tag{2}$$

$$\frac{\dot{P}}{P} = \gamma(Y - \overline{Y}) \tag{3}$$

$$\frac{\dot{S}}{S} = i - i^* \tag{4}$$

- For given values of $P$ and $S$, equations (1) and (2) define a temporary equilibrium for $Y$ and $i$ (IS-LM model)

$$\begin{aligned} Y &= Y\left(\frac{SP^*}{P}, Y^*\right) \\ i &= i\left(\frac{M}{P}, \frac{SP^*}{P}, Y^*\right) \end{aligned}$$

- Substitute into Phillips curve (3) and UIP condition (4)

$$\begin{aligned} \frac{\dot{P}}{P} &= \gamma\left(Y\left(\frac{SP^*}{P}, Y^*\right) - \overline{Y}\right) \\ \frac{\dot{S}}{S} &= i\left(\frac{M}{P}, \frac{SP^*}{P}, Y^*\right) - i^* \end{aligned}$$

- Compact notation

$$\begin{aligned} \dot{P} &= \phi_1(P, S; P^*, Y^*) \\ \dot{S} &= \phi_2(P, S; P^*, Y^*, M, i^*) \end{aligned}$$

- To analyse the stability of the system we calculate the Jacobian (NOTE! EVALUATED IN THE STATIONARY EQUILIBRIUM $Y = \overline{Y}$ and $i = i^*$)

$$
\begin{aligned}
\frac{\partial \phi_1}{\partial P} &= \frac{\partial \dot{P}}{\partial P} = P\gamma \frac{dY}{dP} = -\gamma \frac{SP^*}{P} \frac{X_R}{1 - C_Y - X_Y} < 0 \\
\frac{\partial \phi_1}{\partial S} &= \frac{\partial \dot{P}}{\partial S} = P\gamma \frac{dY}{dS} = \gamma \frac{X_R}{1 - C_Y - X_Y} P^* > 0 \\
\frac{\partial \phi_2}{\partial P} &= \frac{\partial \dot{S}}{\partial P} = S \frac{di}{dP} = S \frac{M}{P^2} \frac{1}{m_i} \left( El_Y \left( \frac{M}{P} \right) \times El_R Y - 1 \right) \lesseqqgtr 0 \\
\frac{\partial \phi_2}{\partial S} &= \frac{\partial \dot{S}}{\partial S} = S \frac{di}{dS} = -\frac{SP^*}{P} \frac{m_Y}{m_i} \frac{X_R}{1 - C_Y - X_Y} > 0
\end{aligned}
$$

- The determinant of $A$ is

$$
\begin{aligned}
|A| &= \frac{\partial \phi_1}{\partial P}\frac{\partial \phi_2}{\partial S} - \frac{\partial \phi_1}{\partial S}\frac{\partial \phi_2}{\partial P} \\
&= \gamma \frac{SP^*}{P}\frac{X_R}{1 - C_Y - X_Y}\frac{SP^*}{P}\frac{m_Y}{m_i}\frac{X_R}{1 - C_Y - X_Y} \\
&\quad -\gamma \frac{X_R}{1 - C_Y - X_Y} SP^* \left( \frac{m_Y}{m_i}\frac{SP^*}{P^2}\frac{X_R}{1 - C_Y - X_Y} - \frac{M}{P^2}\frac{1}{m_i} \right) \\
&= \gamma \frac{m_Y}{m_i}\left( \frac{SP^*}{P}\frac{X_R}{1 - C_Y - X_Y} \right)^2 - \gamma \frac{m_Y}{m_i}\left( \frac{SP^*}{P}\frac{X_R}{1 - C_Y - X_Y} \right)^2 \\
&\quad +\gamma \frac{X_R}{1 - C_Y - X_Y}\frac{SP^*M}{P^2}\frac{1}{m_i} \\
&= \gamma \frac{X_R}{1 - C_Y - X_Y}\frac{SP^*M}{P^2}\frac{1}{m_i} < 0
\end{aligned}
$$

- The system exhibits (local) saddle point stability: for a given initial value of $P$, we can always find an initial value $S$ which gets us onto a path which approaches the steady state as $t \to \infty$

- The initial exchange rate is pinned down by assuming that $S$ jumps immediately to this saddle path.

# Phase diagram

- Recall stationary equilibrium

$$\dot{P} = 0 \Leftrightarrow Y\left(\frac{SP^*}{P}, Y^*\right) = \overline{Y}$$
$$\dot{S} = 0 \Leftrightarrow i\left(\frac{M}{P}, \frac{SP^*}{P}, Y^*\right) = i^*$$

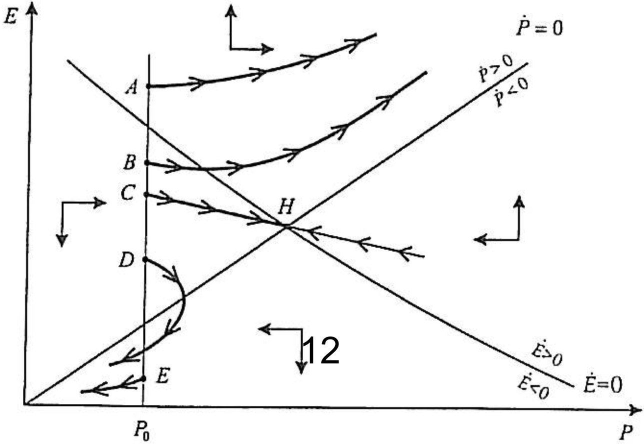
- Slope of $\dot{P} = 0$ curve in $(P, S)$ space

$$\left.\frac{dS}{dP}\right|_{\dot{P}=0} = -\frac{dY/dP}{dY/dS} = \frac{S}{P} > 0$$

- Slope of $\dot{S} = 0$ curve in $(P, S)$ space

$$\left.\frac{dS}{dP}\right|_{\dot{S}=0} = -\frac{di/dP}{di/dS} \lesseqqgtr 0$$

- Baseline scenario (overshooting): $di/dP > 0 \rightarrow \dot{S} = 0$ is downward sloping


E
$\dot{P}=0$
$\dot{P}>0$
$\dot{P}<0$
A
B
C
H
D
E
$\dot{E}>0$
$\dot{E}<0$
$\dot{E}=0$
$P_0$
P

# An unanticipated permanent increase in the money supply

- New stationary equilibrium $Y = \overline{Y}$ and $i = i^*$:
  - $R$ and $Y$ will be unchanged
  - If $R$ is unchanged, $M/P$ must be unchanged
  - Hence; the percentage increase in $P$ and $S$ is equal to the percentage increase in $M$ in the long run

- The model satisfies long-run monetary neutrality

- Graphical analysis
  - The $\dot{S} = 0$ curve shifts to the right

$$\left.\frac{dS}{dM}\right|_{\dot{S}=0,\ P \text{ constant}} = -\frac{di/dM}{di/dS} > 0$$

  - The $\dot{P} = 0$ locus is unaffected (aggregate demand is independent of the money supply)
  - The exchange rate jumps immediately to the new saddle path

- Overshooting result:
  - Exchange rate jumps immediately to the new saddle path
  - Depreciation increases $Y$ and the demand for money
  - Since domestic prices are sticky $M/P$ increases and $i$ falls (since we have assumed that $El_Y\left(\frac{M}{P}\right) \times El_R Y < 1$)
  - Negative interest differential must be offset by $\dot{S}/S < 0$ which can only happen if the exchange rate initially increases above its new long-run equilibrium value (i.e., overshoots the final effect)
  - The initial depreciation of the exchange rate produces excess demand for goods causes prices to increase.
  - $Y$ and $i$ then fall back to their original equilibrium levels
- Key assumption generating overshooting result: exchange rates and asset markets adjust fast relative to goods markets.

- Is overshooting a general result?

- No! If $El_Y\left(\frac{M}{P}\right) \times El_R Y > 1$

  – the $\dot{S} = 0$ locus is upward sloping

  – note that $\dot{P} = 0$ locus is steeper than the $\dot{S} = 0$ locus

$$
\begin{aligned}
\left.\frac{dS}{dP}\right|_{\dot{P}=0} - \left.\frac{dS}{dP}\right|_{\dot{S}=0} &= \frac{S}{P} - \frac{\frac{m_Y}{m_i}\frac{SP^*}{P^2}\frac{X_R}{1-C_Y-X_Y} - \frac{M}{P^2}\frac{1}{m_i}}{\frac{m_Y}{m_i}\frac{P^*}{P}\frac{X_R}{1-C_Y-X_Y}} \\
&= \frac{\frac{S}{P}\frac{m_Y}{m_i}\frac{P^*}{P}\frac{X_R}{1-C_Y-X_Y} - \frac{m_Y}{m_i}\frac{SP^*}{P^2}\frac{X_R}{1-C_Y-X_Y} + \frac{M}{P^2}\frac{1}{m_i}}{\frac{m_Y}{m_i}\frac{P^*}{P}\frac{X_R}{1-C_Y-X_Y}} \\
&= \frac{\frac{M}{P}}{m_Y P^* \frac{X_R}{1-C_Y-X_Y}} > 0
\end{aligned}
$$

  - we get ‘undershooting’ (i.e., the initial jump in the exchange rate is smaller than the long-run change following an unanticipated permanent increase in the money supply)

- Some additional points

  - Permanent nominal shocks (e.g., shocks to the money supply) do **not** affect the real exchange rate in the long-run

  - Permanent real shocks (e.g., shocks to foreign output) do affect the real exchange rate in the long run

# Empirical evidence

- Model is consistent with a high correlation between nominal and real exchange rates

- Model is consistent with observed real appreciations and high real interest rates observed after large (dramatic) monetary tightenings

- Famous paper (recommended reading!) Meese, R. and K. Rogoff (1983): Empirical exchange rate models of the seventies. Do they fit out of sample?, Journal of International Economics, vol 14 3–24
  - Analyse out of sample forecast performance of monetary models of exchange rate determinantion (including the flexible-price model and the Dornbusch model) and time-series models
  - For major nominal exchange rates against the US dollar (dollar/pound, dollar/mark, dollar/yen + trade weighted dollar exchange rates), the structural models are outperformed by a random walk model at one to twelve month horizons
  - Result holds even if forecasts are based on the actual realised values of future explanatory variables
- Structural models may outperform random walk at longer horizons (two to three years)

Table 1

Root mean square forecast errors.[a]

| Exchange rate | Model: / Horizon | Random walk | Forward rate | Univariate autoregression | Vector autoregression | Frenkel–Bilson[b] | Dornbusch–Frankel[b] | Hooper–Morton[b] |
|---|---|---|---|---|---|---|---|---|
| $/mark | 1 month | 3.72 | 3.20 | 3.51 | 5.40 | 3.17 | 3.65 | 3.50 |
| | 6 months | 8.71 | 9.03 | 12.40 | 11.83 | 9.64 | 12.03 | 9.95 |
| | 12 months | 12.98 | 12.60 | 22.53 | 15.06 | 16.12 | 18.87 | 15.69 |
| $/yen | 1 month | 3.68 | 3.72 | 4.46 | 7.76 | 4.11 | 4.40 | 4.20 |
| | 6 months | 11.58 | 11.93 | 22.04 | 18.90 | 13.38 | 13.94 | 11.94 |
| | 12 months | 18.31 | 18.95 | 52.18 | 22.98 | 18.55 | 20.41 | 19.20 |
| $/pound | 1 month | 2.56 | 2.67 | 2.79 | 5.56 | 2.82 | 2.90 | 3.03 |
| | 6 months | 6.45 | 7.23 | 7.27 | 12.97 | 8.90 | 8.88 | 9.08 |
| | 12 months | 9.96 | 11.62 | 13.35 | 21.28 | 14.62 | 13.66 | 14.57 |
| Trade-weighted dollar | 1 month | 1.99 | N.A. | 2.72 | 4.10 | 2.40 | 2.50 | 2.74 |
| | 6 months | 6.09 | N.A. | 6.82 | 8.91 | 7.07 | 6.49 | 7.11 |
| | 12 months | 8.65 | 14.24 | 11.14 | 10.96 | 11.40 | 9.80 | 10.35 |

[a]Approximately in percentage terms.
[b]The three structural models are estimated using Fair's instrumental variable technique to correct for first-order serial correlation.

$$\text{root mean square error} = \left\{ \sum_{s=0}^{N_k - 1} [F(t+s+k) - A(t+s+k)]^2 / N_k \right\}^{1/2}, \quad \text{(3c)}$$